

Emergence of Space and Spacetime Dynamics of Friedmann-Robertson-Walker Universe

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Abstract

In a recent paper [arXiv:1206.4916] by T. Padmanabhan, it was argued that our universe provides an ideal setup to stress the issue that cosmic space is emergent as cosmic time progresses and that the expansion of the universe is due to the difference between the number of degrees of freedom on a holographic surface and the one in the emerged bulk. In this note following this proposal we obtain the Friedmann equation of a higher dimensional Friedmann-Robertson-Walker universe. By properly modifying the volume increase and the number of degrees of freedom on the holographic surface from the entropy formulas of black hole in the Gauss-Bonnet gravity and more general Lovelock gravity, we also get corresponding dynamical equations of the universe in those gravity theories.

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It is generally believed that our spacetime is emergent. However, it is clearly quite difficult to build spacetime structure from some suitably defined non-geometric variables. On the other hand, according to the equivalence principle, gravity is just the dynamics of spacetime. This implies that gravity is also an emergent phenomenon. Indeed, the idea that gravity appears as an emergent phenomenon can date back to the proposal made by Sakharov in 1967 [1]. In this so-called induced gravity, spacetime background emerges as a mean field approximation of some underlying microscopic degrees of freedom, similar to hydrodynamics or continuum elasticity theory from molecular physics [2]. Various aspects of the recent research on the relation between thermodynamics and gravitational dynamics support such a point of view (for a review, see [3]).

In the studies of the connection between thermodynamics and gravitational dynamics, a lot of attention is paid on how the gravitational field equations appear from the thermodynamical viewpoint. In 1995, Jacobson [4] derived Einstein's field equations by employing the fundamental Clausius relation $\delta Q = TdS$ together with the equivalence principle. Here the key idea is to demand that this relation holds for all the local Rindler causal horizon through each spacetime point, with δQ and T interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this way, Einstein's equation is nothing but an equation of state of spacetime. The Clausius relation also arises in the interpretation of gravitational field equations as an entropy balance law, $\delta S_m = \delta S_{\text{grav}}$, across a null surface [5].

In a paper by Verlinde [6], the viewpoint of gravity being not a fundamental interaction has been further advocated. Gravity is explained as an entropic force caused by changes in the information associated with the positions of material bodies. With the assumption of the entropic force together with the Unruh temperature [7], Verlinde is able to derive the second law of Newton; with the assumption of the entropic force together with the holographic principle and the equipartition law of energy he obtains the Newton's law of gravitation. Similar observations were also made by Padmanabhan [8]. He observed that the equipartition law of energy for the horizon degrees of freedom combining with the thermodynamic relation $S = E/2T$, also leads to the Newton's law of gravity, here S and T are thermodynamic entropy and temperature associated with the horizon and E is the active gravitational mass producing the gravitational acceleration in the spacetime [9].

In the setting of a Friedmann-Robertson-Walker (FRW) universe, applying the Clausius relation to the apparent horizon of the universe with any spatial curvature, one is able to derive the Friedmann equations describing the dynamics of the universe, not only in the Einstein's general relativity, but also in the Gauss-Bonnet gravity and more general Lovelock gravity [10]. In this derivation, one assumes that there is a Hawking temperature

$T = 1/2\pi r_A$ associated with the apparent horizon [11], here r_A is the apparent horizon radius. While in the entropic force picture of gravity, combining the holographic principle with the equipartition law of energy and the Unruh temperature, one also can get the Friedmann equations of the FRW universe [12], in which the Komar mass plays the role as the source to produce the gravitational field.

Note that in these studies, one discusses the gravitational field equations as the equations of emergent phenomenon, assuming the existence of spacetime manifold. Clearly it is quite difficult to stress the issue for “spacetime itself as an emergent structure”. Time is a parameter to describe the evolution of some dynamical variables. It is therefore even more difficult conceptually to think of time as being emergent from some pre-geometric variables, compared to space. Very recently, Padmanabhan [13] argued that our universe provides a setup to stress the issue that *the cosmic space is emergent as the cosmic time progresses*, because the cosmic time of a geodesic observer plays a special role, to which the cosmic microwave background radiation is homogeneous and isotropic. He argued that the expansion of the universe is due to the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of emerged space and successfully derived the dynamical equation of a FRW universe. In this paper we would like to further elaborate on this issue and generalize to the Gauss-Bonnet gravity and more general Lovelock gravity.

Let us start with the Padmanabhan’s observation. He notices that for a pure de Sitter universe with Hubble constant H , the holographic principle can be expressed in terms of the form

$$N_{\text{sur}} = N_{\text{bulk}}, \quad (1)$$

where N_{sur} denotes the number of degrees of freedom on the spherical surface with Hubble radius $1/H$, $N_{\text{sur}} = 4\pi/L_p^2 H^2$, where L_p is the Planck length, while the bulk degrees of freedom obey the equipartition law of energy, $N_{\text{bulk}} = |E|/(1/2)T$, in this paper we set $k_B = 1$. If one takes the temperature to be the Hawking temperature $T = H/2\pi$ associated with the Hubble horizon and E to be the Komar energy $|(\rho + 3p)|V$ contained inside the Hubble volume $V = 4\pi/3H^3$, and considers $p = -\rho$ for the de Sitter space, one immediately from (1) has $H^2 = 8\pi L_p^2 \rho/3$, which shows the consistence between (1) and Einstein’s field equations.

With the equality (1), one has $|E| = (1/2)N_{\text{sur}}T$, which is the standard equipartition law. This equipartition law in the gravity context was first found for static spacetime in [9]. In the entropic force picture, the equipartition law is also used to describe the degrees of freedom on the holographic screen [6]. In [13], Padmanabhan called the equality (1) the *holographic equipartition* because it relates the effective degrees of freedom residing in

the bulk to the degrees of freedom on the boundary surface.

The validity of the holographic equipartition (1) for the pure de Sitter leads Padmanabhan to further consider our real universe, which is asymptotically de Sitter, as shown by a lot of astronomical observations. The main idea is to regard that the expansion of the universe, conceptually equivalent to the emergence of space, is being derived towards the holographic equipartition, and the basic law governing the emergence of space must relate the emergence of space to the different between the number of degrees of freedom in the holographic surface and the one in the emerged bulk [13]. He proposed that in an infinitesimal interval dt of cosmic time, the increase dV of the cosmic volume is given by

$$\frac{dV}{dt} = L_p^2(N_{\text{sur}} - N_{\text{bulk}}), \quad (2)$$

Putting the cosmic volume $V = 4\pi/3H^3$, the degrees of freedom on the holographic boundary $N_{\text{sur}} = 4\pi/L_p^2 H^2$, the temperature $T = H/2\pi$, and the degrees of freedom in the bulk N_{bulk} given in terms of the Komar energy in the bulk, he arrived at

$$\frac{\ddot{a}}{a} = -\frac{4\pi L_p^2}{3}(\rho + 3p). \quad (3)$$

This is nothing, but the standard dynamical equation of a FRW universe filled by perfect fluid with energy density ρ and pressure p . With the continuity equation, $\dot{\rho} + 3H(\rho + p) = 0$, integrating (3), one can get the Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi L_p^2}{3}\rho, \quad (4)$$

where k is an integration constant, which can be interpreted as the spatial curvature of the FRW universe.

One can see from (1) and (2) the necessary of the existence of the cosmological constant in order to have the asymptotic holographic equipartition [13]. The existence of the cosmological constant derives our universe towards the state with the holographic equipartition. Clearly without the cosmological constant, such a state cannot be reached. In this note we will not further discuss the implications of (2) for dark energy, instead we will pay attention on the dynamical equations of the universe from (2).

We now first generalize the above derivation process of the dynamical equation of the universe to the higher $(n + 1)$ -dimensional case with $n > 3$. In this case the number of degrees of freedom on the holographic surface is given by [6]

$$N_{\text{sur}} = \alpha A/L_p^{n-1}, \quad (5)$$

where $A = n\Omega_n/H^{n-1}$ and $\alpha = (n-1)/2(n-2)$ with Ω_n being the volume of an n -sphere with unit radius. In this case, we make a minor modification for the proposal (2) as

$$\alpha \frac{dV}{dt} = L_p^{n-1} (N_{\text{sur}} - N_{\text{bulk}}), \quad (6)$$

where the volume $V = \Omega_n/H^n$. In the case of $(n+1)$ -dimensions, the bulk Komar energy is [12]

$$E_{\text{Komar}} = \frac{(n-2)\rho + np}{n-2} V, \quad (7)$$

and then the bulk degrees of freedom read

$$N_{\text{bulk}} = -\frac{2E_{\text{Komar}}}{T}, \quad (8)$$

where we have added a minus sign in front of E_{Komar} , in order to have $N_{\text{bulk}} > 0$, which makes sense only in the accelerating phase with $(n-2)\rho + np < 0$ [13]. For normal matters, one needs not the minus sign. Substituting (5) and (8) into (6) with temperature $T = H/2\pi$, we obtain

$$\frac{\ddot{a}}{a} = -\frac{8\pi L_p^{n-1}}{n(n-1)} [(n-2)\rho + np], \quad (9)$$

With the continuity equation in $(n+1)$ dimensions, $\dot{\rho} + nH(\rho + p) = 0$, integrating (9) yields the standard Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{16\pi L_p^{n-1}}{n(n-1)} \rho. \quad (10)$$

It is direct to see that when $n = 3$, our equations (9) and (10) reduce to the ones (3) and (4), respectively. Here once again, k is an integration constant.

Note that (9) and (10) are dynamical equations of a FRW universe in the Einstein's general relativity. It is therefore quite interesting to see whether the above procedure works or not in the Gauss-Bonnet gravity and more general Lovelock gravity, the latter is a natural generalization of general relativity in higher dimensional spacetime. It is well-known that the entropy formula of black hole in the Gauss-Bonnet gravity no longer obeys the Bekenstein-Hawking area formula, instead it has an additional correction term [14]

$$S = \frac{A_+}{4L_p^{n-1}} \left(1 + \frac{n-1}{n-3} \frac{2\tilde{\alpha}}{r_+^2} \right), \quad (11)$$

where $A_+ = n\Omega_n r_+^{n-1}$ is the horizon area of the black hole with horizon radius r_+ and $\tilde{\alpha}$ is the Gauss-Bonnet coefficient with dimension [length]². Applying the entropy formula

(11) to the holographic surface in our setup, we assume that the effective area of the holographic surface is

$$\tilde{A} = A \left(1 + \frac{n-1}{n-3} \frac{2\tilde{\alpha}}{H^{-2}} \right), \quad (12)$$

with $A = n\Omega_n/H^{n-1}$. Based on the relation between the volume V and the area A of an n -sphere with radius R , one has

$$\frac{dV}{dA} = \frac{R}{n-1}. \quad (13)$$

Then we further assume that the effective volume increase in the case of Gauss-Bonnet gravity satisfies

$$\frac{d\tilde{V}}{dt} = \frac{1}{(n-1)H} \frac{d\tilde{A}}{dt}, \quad (14)$$

$$= -\frac{n\Omega_n}{H^{n+1}} (1 + 2\tilde{\alpha}H^2) \dot{H}, \quad (15)$$

$$= -\frac{n\Omega_n}{2H^{n+2}} (H^2 + \tilde{\alpha}H^4). \quad (16)$$

We suppose from (16) that the number of degrees of freedom on the holographic surface is given by

$$N_{\text{sur}} = \alpha \frac{n\Omega_n}{H^{n+1}L_p^{n-1}} (H^2 + \tilde{\alpha}H^4), \quad (17)$$

and in this case, the bulk degrees of freedom is still given by (8). Putting (15) and (17) into (6), we obtain

$$(1 + 2\tilde{\alpha}H^2) \dot{H} + (1 + \tilde{\alpha}H^2) H^2 = -\frac{8\pi L_p^{n-1}}{n(n-1)} [(n-2)\rho + np]. \quad (18)$$

Integrating (18) with the continuity equation, we arrive at

$$H^2 + \tilde{\alpha}H^4 = \frac{16\pi L_p^{n-1}}{n(n-1)} \rho. \quad (19)$$

Here we have set the integration constant to zero. This is nothing, but corresponding Friedmann equation of spatial flat FRW universe in the Gauss-Bonnet gravity [10]. Note that if one does not set the integration constant to zero, unlike the case in the Einstein's general relativity, it cannot be explained as the spatial curvature of the universe.

In the more general Lovelock gravity, the entropy of black hole has the following form [15]

$$S = \frac{A_+}{4L_p^{n-1}} \sum_{i=1}^m \frac{i(n-1)}{(n-2i+1)} \hat{c}_i r_+^{2-2i}, \quad (20)$$

where \hat{c}_i are some coefficients in front of Euler density terms in the theory and $m = [n/2]$. We suppose from the entropy expression that the effective area of the holographic surface is

$$\tilde{A} = \frac{n\Omega_n}{H^{n-1}} \sum_{i=1}^m \frac{i(n-1)}{(n-2i+1)} \hat{c}_i H^{2i-2}, \quad (21)$$

the increase of the effective volume is then given by

$$\frac{d\tilde{V}}{dt} = \frac{1}{(n-1)H} \frac{d\tilde{A}}{dt}, \quad (22)$$

$$= -\frac{n\Omega_n}{H^{n+3}} \left(\sum_{i=1}^m i \hat{c}_i H^{2i} \right) \dot{H}, \quad (23)$$

$$= -\frac{n\Omega_n}{2H^{n+2}} \left(\sum_{i=1}^m \hat{c}_i H^{2i} \right). \quad (24)$$

In this case, we assume from (24) that the number of the degrees of freedom on the holographic surface is

$$N_{\text{sur}} = \alpha \frac{n\Omega_n}{H^{n+1} L_p^{n-1}} \sum_{i=1}^m \hat{c}_i H^{2i}. \quad (25)$$

Substituting (23) and (25) into (6) together with the bulk degrees of freedom (8), we arrive at

$$\left(\sum_{i=1}^m i \hat{c}_i H^{2i-2} \right) \dot{H} + \sum_{i=1}^m \hat{c}_i H^{2i} = -\frac{8\pi L_p^{n-1}}{n(n-1)} [(n-2)\rho + np]. \quad (26)$$

Integrating this equation with help of the continuity equation yields

$$\sum_{i=1}^m \hat{c}_i H^{2i} = \frac{16\pi L_p^{n-1}}{n(n-1)} \rho, \quad (27)$$

where we have also set the integration constant to vanish. This equation is just corresponding Friedmann equation of a flat FRW universe in the Lovelock gravity [10]. Note that here like the case of the Gauss-Bonnet gravity, the integration constant cannot be interpreted as the spatial curvature of the universe.

To summarize, in this short note we have investigated the idea recently made by Padmanabhan [13] that the emergence of space and expansion of the universe are due to the difference between the number of degrees of freedom on the holographic surface and the one in the emerged bulk. In the setup of a higher dimensional FRW universe, we have indeed derived the dynamical equations of the universe. From the entropy formulas of black hole in the Gauss-Bonnet gravity and more general Lovelock gravity, by properly modifying the volume increase of the emerged space and the number of degrees of freedom on the holographic surface, we have also obtained correct Friedmann equations of a flat

FRW universe in those gravity theories. Finally we would like to mention two points here. One is that in the case of Einstein's general relativity, from the acceleration equation (9) to the Friedmann equation (10), the integration constant k can be interpreted as the spatial curvature of the FRW universe, but it is no longer valid in the cases of Gauss-Bonnet gravity and Lovelock gravity. The other is that in the setting of this paper the expression of the number of degrees of freedom on the holographic surface is not simply the same as the one of black hole entropy, although they closely relate each other, in the Gauss-Bonnet gravity and Lovelock gravity. These are worthy further to investigate.

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